Student Name and ID Number

MATH 3012 Final Exam, May 3, 2007, WTT

1. Acme Manufacturing Company makes computers and assigns each cpu they produce an indentification number (string) consisting of four capital letters, followed by a string of 12 digits. The digits are divided into two blocks, one of size 4 and the second of size 8. Two dashes are used, one to separate the letters from the digits, and the second to separate the two blocks of digits. For example, a typical ID number might be: XTBM-9300-14000327.

a. How many distinct ID numbers are possible?

b. How many ID numbers have three A's, one B, four 0's, three 5's and five 7's?

c. Of the ID numbers in part b, how many have the five 7's occuring consecutively in the second block of digits, e.g., ABAA-5005-07777705 is one such ID number?

2. How many integer valued solutions to the following equations and inequalities:

a. $x_1 + x_2 + x_3 = 73$, all $x_i > 0$.

b. $x_1 + x_2 + x_3 = 73$, all $x_i \geq 0$.

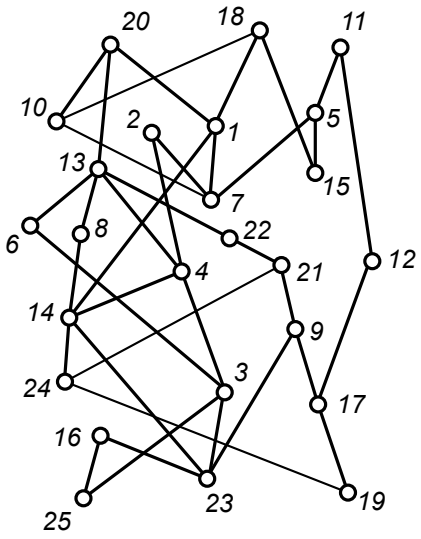
c. $x_1 + x_2 + x_3 \leq 73$, all $x_i > 0$.

d. $x_1 + x_2 + x_3 \leq 73$, all $x_i > 0$, $x_3 < 26$.

3. Use the Euclidean algorithm to find $d = \gcd(5544, 1575)$.

4. Use your work in the preceding problem to find integers x and y so that $d = 5544x + 1575y$.

5.



a. Find the set of minimal elements of this poset.

b. How many elements of are comparable with the point labeled 4?

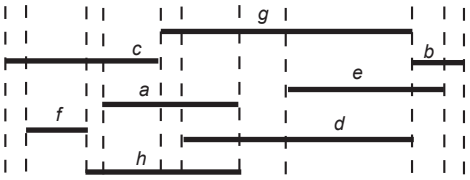
c. Explain why $\{3, 6, 13, 20, 23\}$ is a maximal chain.

d. For each x, let height(x) denote the maximum size of a chain having x as its greatest element. In particular, height$(x) = 1$ if and only if x is a minimal element. Writing directly on the diagram, label each point with the integer representing its height.

e. Find the height h of this poset

f. Find a chain of h points.

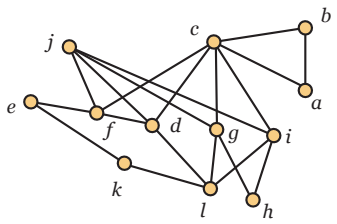
6. Define an interval order P with point set $X = \{a, b, c, d, e, f, g, h, i, j\}$. by the following interval representation. This representation uses 11 different endpoints.



a. Find the number d of distinct down sets, the number u of distinct upsets and verify that $d = u$. Then find the unique interval representation of P using intervals with endpoints from $\{1, 2, \ldots, d\}$

b. Use the First Fit algorithm to a partition of this poset into a minimum number of chains. Provide your answer by labeling the intervals in the diagram with positive integers so that all elements assigned the same integer form a chain. Then find a maximum antichain in this poset.

7.



a. Use the Greedy Algorithm and alphabetic order to find an euler circuit in the graph above. Your answer should be given as a sequence of partial circuits starting with the trivial circuit (a).

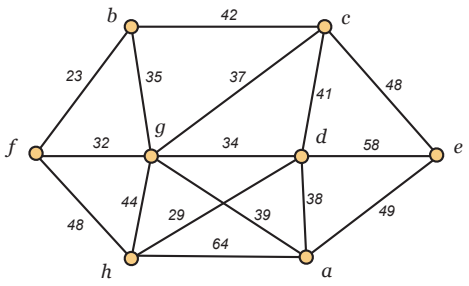
b. Show that the chromatic number of the graph above is 3 by labeling each vertex in the diagram with an integer from $\{1, 2, 3\}$ so that all vertices with the same label form an independent set.

c. Find a maximum clique in the graph above.

d. Find a maximum cycle in the graph above.

e. Explain why this graph is not perfect.

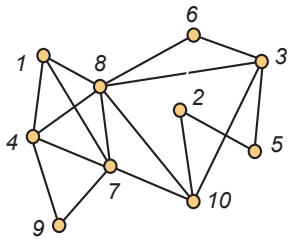
8.



In the space below, list *in order* the edges which make up a minimum weight spanning tree using, respectively Kruskal's Algorithm (avoid cycles) and Prim's Algorithm (build tree). For Prim, use vertex a as the root.

Kruskal's Algorithm **Prim's Algorithm**

9.



a. Show that this graph is hamiltonian by listing the vertices in an order which forms a cycle of size 10.

b. Explain why this graph does not have an euler circuit—but does have an euler path.

10. A data file `digraph_data.txt` has been read for a digraph whose vertex set is [7]. The weights on the directed edges are shown in the matrix below. Apply Dijkstra's algorithm to find the distance from vertex 1 to all other vertices in the graph. Also, for each x, find a shortest path from 1 to x.

W	1	2	3	4	5	6	7
1	0	28	13	41	62	52	98
2	60	0	28	9	30	8	44
3	46	60	0	19	42	33	60
4	16	13	17	0	8	14	10
5	23	11	7	13	0	28	1
6	19	8	82	16	10	0	11
7	2	6	3	5	4	9	0

11. Write the general solution of the advancement operator equation:
$(A+2)^4(A-1)^2(A-3)^2(A-5)f = 0$.

12. Find a particular solution to the advancement operator equation:
$(A^2 - 8A + 15)f(n) = 21(2)^n$.

13. Find the unique solution to the advancement operator equation:
$(A^2 - 8A + 15)f(n) = 21(2)^n$ with $f(0) = 9$ and $f(1) = 32$.

14. Write the Inclusion-Exclusion formula for the Euler-ϕ function.

15. Use the formula from the preceding problem to find $\phi(1400)$. Note that $1400 = 8 \times 25 \times 7$.

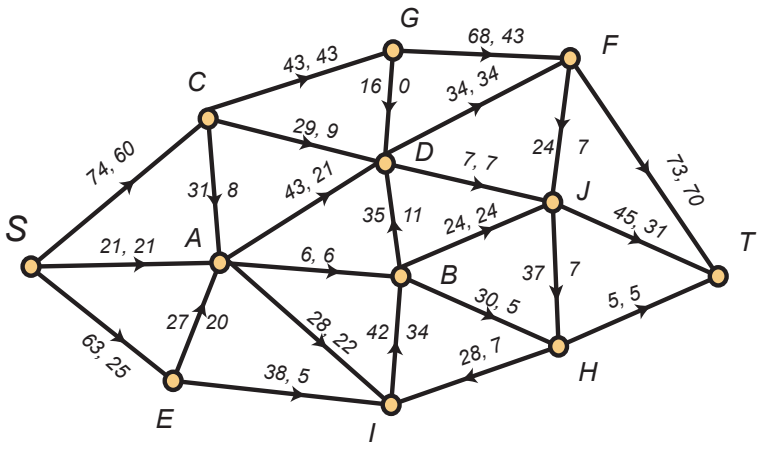
16. For positive integers n and m, let $S(n, m)$ count the number of surjections from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, m\}$. Write the Inclusion-Exclusion formula for $S(n, m)$:

17. Use the formula from the preceding problem to find the value of $S(7, 3)$.

18. Let $R(n, m)$ denote the least positive integer t so that every graph on t vertices contains a complete subgraph of size n or and independent set of size m. Bob claims that it came to him in a dream that $R(5, 7) = 3975$. Alice says Bob's dreams are a suspect source of information. Explain why she is right.

19. What is the formula for the number of labeled trees with vertex set $\{1, 2, \ldots, n\}$?

20.



a. What is the current value of the flow?

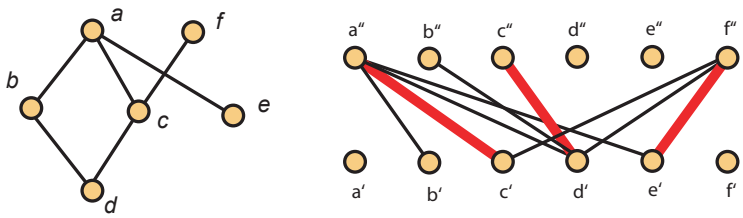
b. What is the capacity of the cut $V = \{S, A, B, C, E, G, H, I\} \cup \{D, F, J, T\}$.

c. Carry out the labeling algorithm, using the pseudo-alphabetic order on the vertices and list below the labels which will be given to the vertices.

d. Use your work in part c to find an augmenting path and make the appropriate changes directly on the diagram.

e. Carry out the labeling algorithm a second time on the updated flow. It should halt without the sink being labeled. Find a cut whose capacity is equal to the value of the flow.

21.



In the figure above, we show a poset and the bipartite graph associated with it. The darkened edges form a maximum matching in the graph. Find the minimum chain partition determined by this matching.