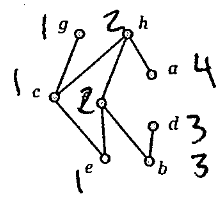
KEY

Student Name and ID Number

MATH 3012 Quiz 2, March 9, 2006, WTT

1.



4 a. This poset is not an interval order. Find four points which induce a copy of $\mathbf{2}+\mathbf{2}$.

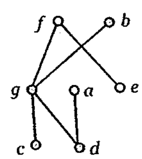
c, e, d, b

4 b. The width of this poset is 4. Find a maximum antichain.

$\{c, f, a, d\}$

4 c. By inspection, find a partition into 4 chains. Give your answer by labeling the points on the diagram with the integers from $\{1, 2, 3, 4\}$ so that all points with the same label form a chain.

2.



5 a. This poset is an interval order. It has 5 distinct down sets. Find them.

$$\underset{D(c)=D(d)=D(e)}{\overset{D_1}{\{\}}} \subseteq \underset{D(a)}{\overset{D_2}{\{d\}}} \subseteq \underset{D(g)}{\overset{D_3}{\{c, d\}}} \subseteq \underset{D(b)}{\overset{D_4}{\{g, c, d\}}} \subseteq \underset{D(f)}{\overset{D_5}{\{e, g, c, d\}}}$$

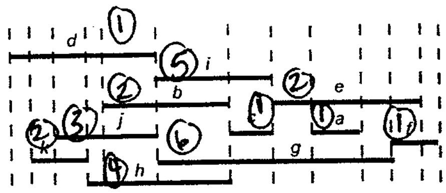
5 b. This poset also has 5 distinct up sets. Find them.

$$\underset{U(d)}{\overset{U_1}{\{a, g, f, b\}}} \supseteq \underset{U(c)}{\overset{U_2}{\{g, f, b\}}} \supseteq \underset{U(g)}{\overset{U_3}{\{f, b\}}} \supseteq \underset{U(e)}{\overset{U_4}{\{f\}}} \supseteq \underset{U(a)=U(f)=U(b)}{\overset{U_5}{\{\}}}$$

5 c. Find the unique interval representation for this poset where every element is assigned an interval with integer endpoints from $\{1, 2, 3, 4, 5\}$.

$I(a) = [2, 5]$
$I(b) = [4, 5]$
$I(c) = [1, 2]$
$I(d) = [1, 1]$
$I(e) = [1, 4]$
$I(f) = [5, 5]$
$I(g) = [3, 3]$

3. Define an interval order P with point set $X = \{a, b, c, d, e, f, g, h, j, k\}$. by the following interval representation.

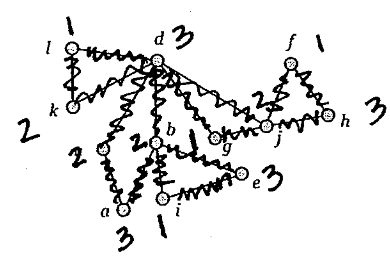


4 a. Use the First Fit algorithm to a partition of this poset into a minimum number of chains. Provide your answer by labeling the intervals in the diagram with positive integers so that all elements assigned the same integer form a chain.

4 b. Find a maximum antichain A in this poset to provide a proof that your partition in part a is indeed minimum.

$\{d, i, b, j, g, h\}$

4.



10 a. Use the Greedy Algorithm described in class to find an euler circuit in this graph. Your answer should be given as a sequence of partial circuits starting with the trivial circuit (a). Proceed by taking the first vertex adjacent to a remaining edge and *always* taking the first available edge—using the alphabetic order.

$(a) \rightarrow (a, b, d, c, a) \rightarrow (a, b, e, i, b, d, c, a) \rightarrow$

(b, e, i, b) $\qquad$ $(d, g, j, f, h, j, d, k, l, d)$

$\rightarrow (a, b, e, i, b, d, g, j, f, h, j, d, k, l, d, c, a)$

4 b. Show that the chromatic number of this graph is 3 by labeling each vertex in the diagram with an integer from $\{1, 2, 3\}$ so that all vertices with the same label form an independent set.

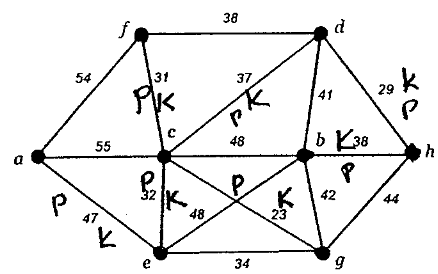
4 c. Find a maximum clique in this graph

Any triangle will do. For example, $\{d, k, l\}$

4 d. Find a maximum cycle in this graph

$\{a, b, d, c\}$

5.



In the space below, list *in order* the edges which make up a minimum weight spanning tree using, respectively Kruskal's Algorithm (avoid cycles) and Prim's Algorithm (build tree). For Prim, use vertex a as the root.

10 Kruskal's Algorithm

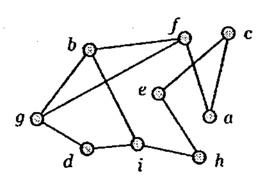
c g 23
d h 29
f c 31
c e 32

c d 37
bh 38
ae 47

10 Prim's Algorithm

ae 47
e c 32
c g 23
f c 31
c d 37
d h 29
h b 38

6.



4 a. Show that this graph is hamiltonian by listing the vertices in an order which forms a cycle of size 9.

(a, c, e, h, i, d, g, b, f)

4 b. Explain why this graph has neither an euler circuit nor an euler path.

It has more than two vertices of odd degree.

15 7. A data file `digraph_data.txt` has been read for a digraph whose vertex set is [7]. The weights on the directed edges are shown in the matrix below. Apply Dijkstra's algorithm to find the distance from vertex 1 to all other vertices in the graph. Also, for each x, find a shortest path from 1 to x.

W	1	2	3	4	5	6	7
1	0	12	51	20	34	5	19
2	60	0	28	8	19	7	24
3	46	60	0	19	9	60	64
4	16	43	17	0	8	14	19
5	23	11	7	13	0	28	22
6	19	6	82	15	28	0	10
7	11	16	23	25	19	8	0

P	1	0	(1)
	2	12	(1,2)
	3	51	(1,3)
	4	20	(1,4)
	5	34	(1,5)
P	6	5	(1,6)
	7	19	(1,7)

P2	11	(1,6,2)
3	51	(1,3)
4	20	(1,4)
5	33	(1,6,5)
7	15	(1,6,7)

3	39	(1,6,2,3)
4	19	(1,6,2,4)
5	30	(1,6,2,5)
P7	15	(1,6,7)

3	38	(1,6,7,3)
P 4	19	(1,6,2,4)
5	30	(1,6,2,5)

3	36	(1,6,2,4,3)
P5	27	(1,6,2,4,5)

P3	34	(1,6,2,4,5,3)