Master

KEY

Student Name and ID Number

MATH 3012 Section F, Quiz 2, March 11, 2009, WTT

8

1. A relation R on a set X is *symmetric* if $(x, y) \in R$ implies $(y, x) \in R$ for all $x, y \in X$. If n is a positive integer and $X = \{1, 2, \ldots, n\}$, how many symmetric relations are there on X?

$$2^n \, 2^{\binom{n}{2}}$$

2. How many equivalence relations are there on the set $\{1, 2, \ldots, 63\}$ with class sizes:

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$$8, 8, 8, 5, 5, 5, 5, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1$$

$$\binom{63}{8,8,8,5,5,5,2,2,2,2,2,2,1,1,1,1,1,1,1} \Big/ 3!\, 4!\, 6!\, 7!$$

3. Count the number of linear extensions of the following poset:

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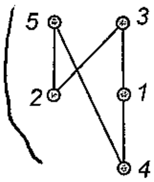
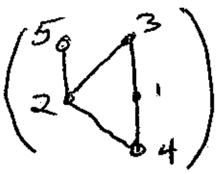
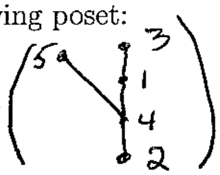
$e\Big($  $\Big) = e\Big($  $\Big) + e\Big($  $\Big) = 5 + 3 = 8$

4. For the subset lattice $\mathbf{2}^{11}$,

12
$= 4 \times 3$

a. The total number of elements is: 2^{11}

b. The total number of maximal chains is: $11!$

c. The number of maximal chains through $\{2, 4, 7, 8\}$ is: $4!\ 7!$

d. The width of $\mathbf{2}^{11}$ is: $\binom{11}{5}$

5. For the following poset:

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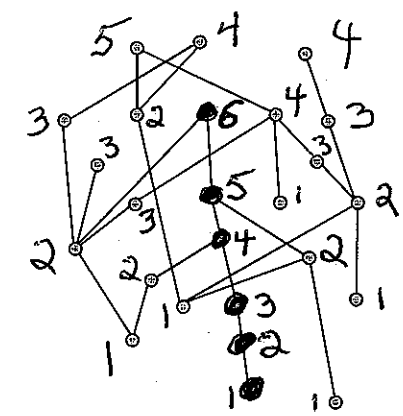

find the height h and a partition into h minimal elements by recursively stripping off the set of minimal elements. You may display your answer by writing directly on the diagram. Then darken a set of points that form a maximum chain.

$h = 6$

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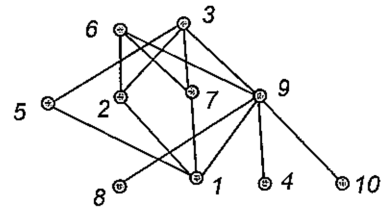
6. The poset **P** shown below is an interval order:



(3) a. Find the elements that are comparable to 8: 9, 6, 3 Note: optional to list 8

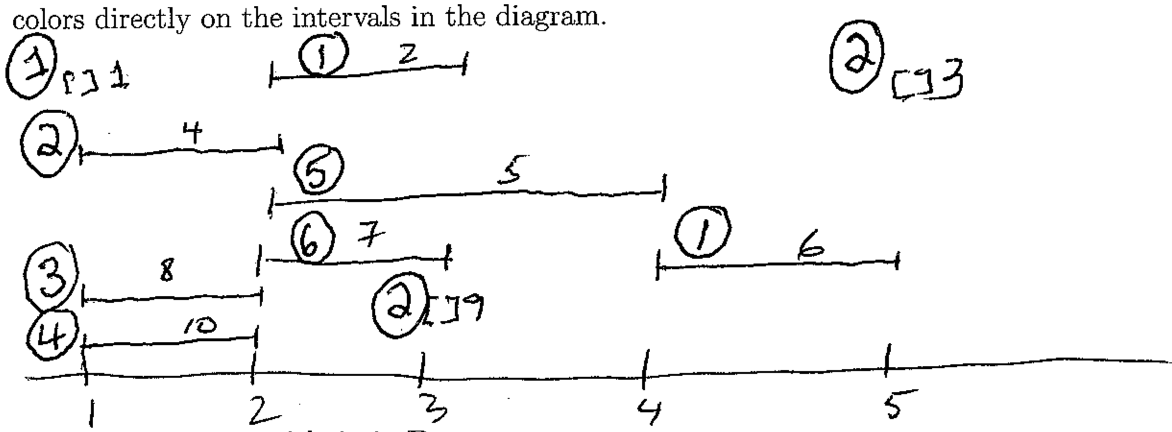
(3) b. Find the elements that are incomparable to 1: 8, 4, 10

(3) c. This poset is ranked; all maximal chains have size 3. What is the maximum rank size: 4

(12) = 3×4 d. Find the down sets and the up sets. Then use these answers to find an interval representation of **P** that uses the least number of end points.

$D(1) = \emptyset$	$U(1) = \{5,2,7,9,6,3\}$	$I(1) = [1,1]$
$D(2) = \{1\}$	$U(2) = \{6,3\}$	$I(2) = [2,3]$
$D(3) = \{5,2,7,9,8,1,4,10\}$	$U(3) = \emptyset$	$I(3) = [5,5]$
$D(4) = \emptyset$	$U(4) = \{9,6,3\}$	$I(4) = [1,2]$
$D(5) = \{1\}$	$U(5) = \{3\}$	$I(5) = [2,4]$
$D(6) = \{2,7,9,8,1,4,10\}$	$U(6) = \emptyset$	$I(6) = [4,5]$
$D(7) = \{1\}$	$U(7) = \{6,3\}$	$I(7) = [2,3]$
$D(8) = \emptyset$	$U(8) = \{9,6,3\}$	$I(8) = [1,2]$
$D(9) = \{8,1,4,10\}$	$U(9) = \{6,3\}$	$I(9) = [3,3]$
$D(10) = \emptyset$	$U(10) = \{9,6,3\}$	$I(10) = [1,2]$

(5) e. In the space below, draw the representation produced in part d. Then use the First Fit Coloring Algorithm for interval graphs to solve the Dilworth Problem for this poset, i.e., find the width w and a partition of **P** into w chains. You may display your answers by writing the colors directly on the intervals in the diagram.



f. Find a maximum antichain in **P**:

width = 6, maximum antichain {4, 2, 5, 7, 8, 10}

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7. Shown below is a diagram for the subset lattice $\mathbf{2}^4$. Show that the graph determined by this diagram is hamiltonian by darkening edges to form a cycle visiting each vertex exactly once.

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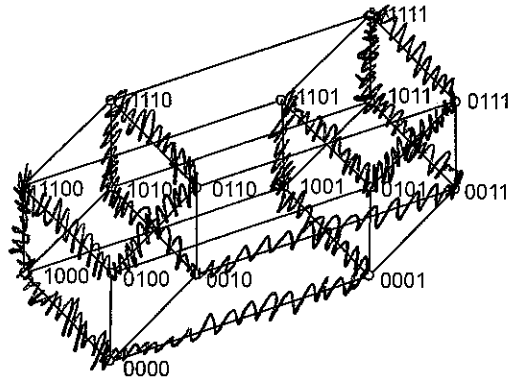

8. $24 = 7+7+3+3+3+1$ is a partition of the integer 24 into "odd parts," while $24 = 16+5+3$ is a partition of the integer 24 into "distinct parts."

a. Write all the partitions of the integer 13 into odd parts:

5

$13 = 13$
$= 11+1+1$
$= 9+3+1$
$= 9+1+1+1+1$
$= 7+5+1$
$= 7+3+1+1+1$
$= 7+3+3$
$= 7+1+1+1+1+1+1$
$= 5+5+3$
$= 5+5+1+1+1$
$= 5+3+3+1+1$
$= 5+3+1+1+1+1+1$
$= 5+1+1+1+1+1+1+1+1$
$= 3+3+3+3+1$
$= 3+3+3+1+1+1+1$
$= 3+3+1+1+1+1+1+1+1$
$= 3+1+1+1+1+1+1+1+1+1+1$
$= 1+1+1+1+1+1+1$
$=$

b. Write all the partitions of the integer 13 into distinct parts (Hint: The number of partitions in part a is the same as the number of partitions in part b. Don't forget that the trivial partition $n = n$ must be considered):

5

$13 = 13$
$= 12+1$
$= 11+2$
$= 10+3$
$= 10+2+1$
$= 9+4$
$= 9+3+1$
$= 8+5$
$= 8+4+1$
$= 8+3+2$
$= 7+6$
$= 7+5+1$
$= 7+4+2$
$= 7+3+2+1$
$= 6+5+2$
$= 6+4+3$
$= 6+4+2+1$
$= 5+4+3+1$

Note! 18 of each type!!!

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Entire Test: $51+31+18=100$