Student Name and ID Number

MATH 3012 Quiz 1, September 18, 2014, WTT

1. Consider the 26-element set consisting of the capital letters of the English alphabet: $\{A, B, C, \ldots, Z\}$.

a. How many strings of length 12 can be formed if repetition of symbols is permitted?

b. How many strings of length 12 can be formed if repetition of symbols is *not* permitted?

c. How many strings of length 12 can be formed using exactly four X's, three Y's and five Z's?

d. How many strings of length 12 can be formed using exactly four X's, three Y's and five Z's if the three $Y's$ are required to occur consecutively in the string?

2. How many lattice paths from $(0, 0)$ to $(24, 31)$ do *not* pass through $(15, 19)$?

3. How many integer valued solutions to the following equations and inequalities:

a. $x_1 + x_2 + x_3 = 42$, all $x_i > 0$.

b. $x_1 + x_2 + x_3 = 42$, all $x_i \geq 0$.

c. $x_1 + x_2 + x_3 < 42$, all $x_i > 0$.

d. $x_1 + x_2 + x_3 \leq 42$, all $x_i \geq 0$.

e. $x_1 + x_2 + x_3 = 42$, all $x_1, x_3 > 0$, $x_2 \geq 7$.

f. $x_1 + x_2 + x_3 = 42$, all $x_1, x_3 > 0$, $0 < x_2 \leq 6$.

4. Use the Euclidean algorithm to find $d = \gcd(420, 245)$.

5. Use your work in the preceding problem to find integers a and b so that $d = 420a + 245b$.

6. For a positive integer n, let t_n count the number of ternary strings of length n that do not contain 102 as a substring. Note that $t_1 = 3$, $t_2 = 9$ and $t_3 = 26$. Develop a recurrence relation for t_n and use it to compute t_4, t_5 and t_6.

7. As illustrated on the white board in class, n circles are placed on the plane so that (1) any two circles in the family intersect in two points, and (2) no three circles have a common point. Let r_n denote the number of regions in the plane determined by these circles. Note that $r_1 = 2$ and $r_2 = 4$. Develop a recurrence for r_n and use it to compute r_3, r_4 and r_5 (do *not* attempt to find these values by drawing pictures).

8. Find the coefficient of $x^4y^7z^{24}$ in $(6x - 5y + 8z^2)^{23}$