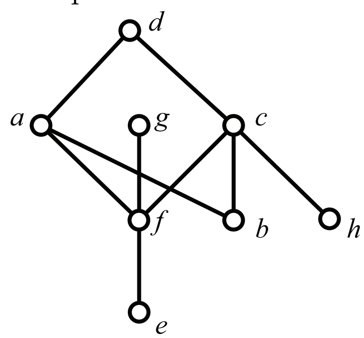
Student Name and ID Number

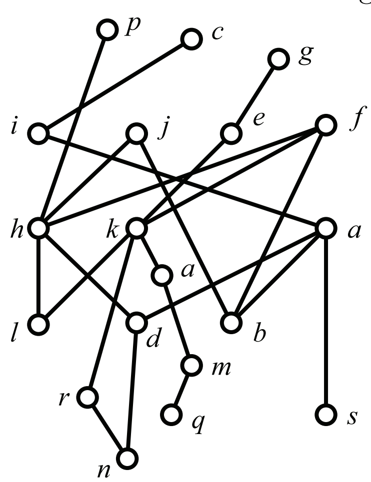
**MATH 3012 Quiz 2, March 15, 2013, WTT**

**1.** Consider the poset shown below. The ground set is $X = \{a, b, c, d, e, f, g, h\}$. In the space to the right of the figure, write the reflexive, antisymmetric and transitive relation on $X$ which defines this poset.




$P =$

**2.** Consider the following poset.




**a.** Find all points comparable to $k$. ____________

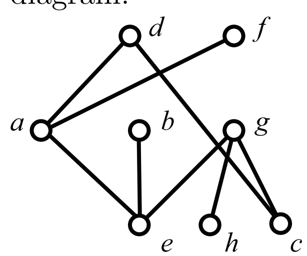
**b.** Find all points which cover $k$. ____________

**c.** Find a maximal chain of size 2. ____________

**d.** Using the algorithm taught in class (recursively removing the set of minimal elements), find the height $h$ of the poset and a partition of $P$ into $h$ antichains. Also find a maximum chain. You may indicate the partition by writing directly on the diagam.

The height $h$ is ______ and ________________ is a maximum chain.

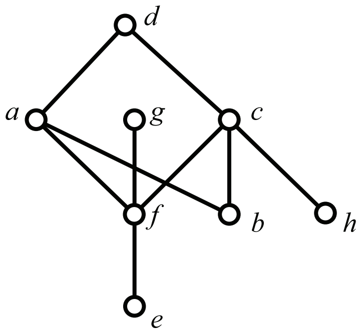
**3.** Find by inspection the width $w$ of the following poset and find a partition of the poset into $w$ chains. Also find a maximum antichain. You may indicate the partition by writing directly on the diagram.



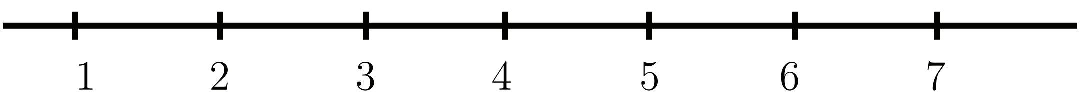

**a**. The width $w$ is ______ and ________________ is a maximum antichain.

**b.** This poset is not an interval order. Find four points which form a copy of $\mathbf{2} + \mathbf{2}$. ________________.

**4.** Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation by computing the down-sets and up-sets in the space provided. Then use the First Fit coloring algorithm to find the width $w$ and a partition of the poset into $w$ chains. Also, find a maximum antichain.




$D(a) =$ $U(a) =$
$D(b) =$ $U(b) =$
$D(c) =$ $U(c) =$
$D(d) =$ $U(d) =$
$D(e) =$ $U(e) =$
$D(f) =$ $U(f) =$
$D(g) =$ $U(g) =$
$D(h) =$ $U(h) =$




The width $w$ is ________ and ____________ is a maximum antichain.

**5.** Let $\mathbf{2}^{15}$ be the poset consisting of all subsets of $\{1, 2, 3, \ldots, 15\}$, ordered by inclusion.

**a.** What is the height of this poset? ________

**b.** What is the width of this poset? ________

**c.** How many maximal chains does the poset have? ________

**d.** How many maximal chains in this poset pass through the set $\{2, 3, 8, 13\}$? ________

**6.** Write the general solution to the homogeneous advancement operator equation:
$\big[A-(7-2i)\big]^3(A-1)^4 f = 0.$

**7.** Find a particular solution to the advancement operator equation:
$(A^2-3A+5)f = 4\cdot 3^n.$

**8.** Write the inclusion-exclusion formula for $S(n,m)$, the number of surjections from $\{1,2,\dots,n\}$ to $\{1,2,\dots,m\}$. Then use this formula to calculate $S(6,4)$.

**9.** Write the inclusion formula for the number $d_n$ of derangements of $\{1,2,\dots,n\}$. Then use this formula to calculate $d_6$.

**10.** Note that $1800 = 25 \cdot 9 \cdot 8$. Use this information and the inclusion-exclusion formula to determine $\phi(1800)$, where $\phi$ is the Euler $\phi$-function studied in class.

**11.** True–False. Mark in the left margin.

1. There is a graph on 928 vertices in which no two vertices have the same degree.
2. There is a poset with 7403 points having width 65 and height 98.
3. There is a poset with 7403 points having width 85 and height 98.
4. The permutation $(8, 1, 4, 9, 3, 6, 2, 7, 5)$ is a derangement.
5. The number of partitions of an integer $n$ into even parts is the same as the number of partitions of $n$ into parts that are all the same.
6. The partitions of a deranged surjection can be effectively computed using inclusion-exclusion and the process will consistently result in a maximum antichain of prime factors.