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Student Name and ID Number

MATH 3012 Quiz 1, September 18, 2014, WTT

1. Consider the 26-element set consisting of the capital letters of the English alphabet: $\{A, B, C, \ldots, Z\}$.

a. How many strings of length 12 can be formed if repetition of symbols is permitted?

b. How many strings of length 12 can be formed if repetition of symbols is *not* permitted?

c. How many strings of length 12 can be formed using exactly four X's, three Y's and five Z's?

d. How many strings of length 12 can be formed using exactly four X's, three Y's and five Z's if the three $Y's$ are required to occur consecutively in the string?

2. How many lattice paths from $(0, 0)$ to $(24, 31)$ do *not* pass through $(15, 19)$?

3. How many integer valued solutions to the following equations and inequalities:

a. $x_1 + x_2 + x_3 = 42, \quad x_1, x_2, x_3 > 0.$

b. $x_1 + x_2 + x_3 = 42, \quad x_1, x_2, x_3 \geq 0.$

c. $x_1 + x_2 + x_3 < 42, \quad x_1, x_2, x_3 > 0.$

d. $x_1 + x_2 + x_3 \leq 42, \quad x_1, x_2, x_3 \geq 0.$

e. $x_1 + x_2 + x_3 = 42, \quad x_1, x_3 > 0,\ x_2 \geq 7.$

f. $x_1 + x_2 + x_3 = 42, \quad x_1, x_3 > 0,\ 0 < x_2 \leq 6.$

4. Use the Euclidean algorithm to find $d = \gcd(420, 245)$.

5. Use your work in the preceding problem to find integers a and b so that $d = 420a + 245b$.

6. For a positive integer n, let s_n count the number of ternary strings of length n that do not contain 102 as a substring. Note that $s_1 = 3$, $s_2 = 9$ and $s_3 = 26$. Develop a recurrence relation for s_n and use it to compute s_4, s_5 and s_6.

7. Let t_n denote the number of ways to tile a $2 \times n$ checkerboard using tiles of the two shapes shown on the white board. One of the shapes is an "L" with a total area of 3. As illustrated, this shape can be used "forwards" and "backwards" but not upside down. The other shape is a 2×1 strip and it can be used vertically or horizontally. Note that $t_1 = 1$, $t_2 = 2$ and $t_3 = 3$. Develop a recurrence for t_n and use it to compute t_4, t_5 and t_6.

8. Find the coefficient of $x^4y^7z^{24}$ in $(6x - 5y + 8z^2)^{23}$

9. True–False. Mark in the left margin.

1. $P(7,3) = 1024$.
2. $C(7,3) = 35$.
3. If 57 pigeons are placed in 7 holes, then there is some hole with at least 9 pigeons.
4. If $f(n) = 865n + 90 \log n$, and $g(n) = 3n + 7$, then $f(n) = O(g(n))$.
5. If $f(n) = 865n + 90 \log n$, and $g(n) = 3n^2 + 7$, then $f(n) = o(g(n))$.
6. $\log n = o(\sqrt{n})$, $\sqrt{n} = o(n)$, $n = o(n^3)$, $n^3 = o(2^n)$, $2^n = o(2^{n^2})$ and $2^{n^2} = o(2^{2^n})$.
7. A recursive permutation tiles non-distinct pigeons with a certificate that can be enumerated but not verified.