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A Generalization of Threshold Graphs with Tolerances

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ABSTRACT

In this extended abstract, we introduce a class of graphs which generalize threshold graphs by introducing threshold tolerances. Several characterizations of these graphs are presented, one of which leads to a polynomial-time recognition algorithm. It is also shown that the complements of these graphs contain interval graphs and threshold graphs, and are contained in the subclass of chordal graphs called strongly chordal graphs, and in the class of interval tolerance graphs. A final paper complete with all proofs will appear at a later time.

1. INTRODUCTION

An undirected graph $G = (V, E)$ is called a threshold tolerance graph if it is possible to associate weights and tolerances with each vertex of G so that two vertices are adjacent exactly when the sum of their weights exceeds either of their tolerances. More formally, there are weights w_v and tolerances t_v for each $v \in V$ so that

$$xy \in E <=> w_x + w_y \geq \min (t_x, t_y). \qquad (*)$$

If we insist that all tolerance be equal, we obtain the class of threshold graphs [CH77]; see also [Go78; Go80, Chapter 10; HZ77; Or77]. It is easy to see that we may require that all weights and tolerances are positive, and that strict inequality holds in (*).

For our purposes, it is convenient to present our results in terms of the complement of threshold tolerance graphs, which we call coTT graphs. An equivalent definition is that a graph $G = (V, E)$ is a coTT graph if there are numbers a_v and b_v for every $v \in V$ so that

$$xy \in E <=> a_x \leq b_y \text{ and } a_y \leq b_x.$$

To see that these definitions are equivalent, set $a_x = w_x$ and $b_x = t_x - w_x$. As before we may take all of these numbers to be positive.

A graph $G = (V, E)$ is called an interval graph [BL76; FG65; GH64; Go80, Chapter 8; LB62] if there are closed intervals $I_v = [L_v, R_v]$ (of the real line) for each $v \in V$ so that two vertices are adjacent exactly when their intervals intersect, that is,

$$xy \in E <=> I_x \cap I_y \neq \emptyset.$$

A graph $G = (V, E)$ is called an interval tolerance graph [GM82, GMT84] if there are intervals $I_v = [L_v, R_v]$ and tolerances τ_v for each $v \in V$ so that